Section 4.3 and 4.4

Math 231

Hope College

Coordinate Representations of Vectors

• Let \mathcal{B} be a finite, ordered basis of a vector space V. Any vector $\mathbf{v} \in V$ can be written uniquely as

$$\alpha_1 \mathbf{x}_1 + \cdots + \alpha_n \mathbf{x}_n$$
.

The vector $[\mathbf{v}]_{\mathcal{B}} = \langle \alpha_1, \dots, \alpha_n \rangle \in \mathbb{R}^n$ is called the **coordinate representation** of \mathbf{v} with respect to the ordered basis \mathcal{B} .

• If V is an n-dimensional vector space and \mathcal{B} is any ordered basis of V, then coordinate representation gives an isomorphism from V to \mathbb{R}^n .



Coordinate Representations of Vectors

• Let \mathcal{B} be a finite, ordered basis of a vector space V. Any vector $\mathbf{v} \in V$ can be written uniquely as

$$\alpha_1 \mathbf{x}_1 + \cdots + \alpha_n \mathbf{x}_n$$
.

The vector $[\mathbf{v}]_{\mathcal{B}} = \langle \alpha_1, \dots, \alpha_n \rangle \in \mathbb{R}^n$ is called the **coordinate representation** of \mathbf{v} with respect to the ordered basis \mathcal{B} .

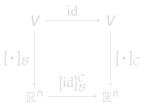
• If V is an n-dimensional vector space and \mathcal{B} is any ordered basis of V, then coordinate representation gives an isomorphism from V to \mathbb{R}^n .



Let *V* be a finite dimensional vector space.

Let $\mathcal{B} = \{\mathbf{x}_1, \dots, \mathbf{x}_n\}$ and $\mathcal{C} = \{\mathbf{y}_1, \dots, \mathbf{y}_n\}$ be bases of V. Let $\mathrm{id}: V \to V$ be the identity function.

- The **transition matrix** matrix $[id]_{\mathcal{B}}^{\mathcal{C}}$ is the $n \times n$ matrix whose j^{th} column is the vector $[\mathbf{x}_j]_{\mathcal{C}}$.
- Theorem 4.30.1: For all $\mathbf{x} \in V$, we have $[\mathrm{id}]_{\mathcal{B}}^{\mathcal{C}}[\mathbf{x}]_{\mathcal{B}} = [\mathbf{x}]_{\mathcal{C}}$.



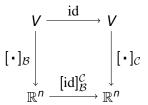
• Theorem 4.30.2: The matrix $[id]_{\mathcal{B}}^{\mathcal{C}}$ is invertible, and $([id]_{\mathcal{B}}^{\mathcal{C}})^{-1} = [id]_{\mathcal{C}}^{\mathcal{B}}$.



Let *V* be a finite dimensional vector space.

Let $\mathcal{B} = \{\mathbf{x}_1, \dots, \mathbf{x}_n\}$ and $\mathcal{C} = \{\mathbf{y}_1, \dots, \mathbf{y}_n\}$ be bases of V. Let $\mathrm{id}: V \to V$ be the identity function.

- The **transition matrix** matrix $[id]_{\mathcal{B}}^{\mathcal{C}}$ is the $n \times n$ matrix whose j^{th} column is the vector $[\mathbf{x}_j]_{\mathcal{C}}$.
- Theorem 4.30.1: For all $\mathbf{x} \in V$, we have $[\mathrm{id}]_{\mathcal{B}}^{\mathcal{C}}[\mathbf{x}]_{\mathcal{B}} = [\mathbf{x}]_{\mathcal{C}}$.



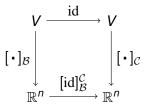
• Theorem 4.30.2: The matrix $[id]_{\mathcal{B}}^{\mathcal{C}}$ is invertible, and $([id]_{\mathcal{B}}^{\mathcal{C}})^{-1} = [id]_{\mathcal{C}}^{\mathcal{B}}$.



Let *V* be a finite dimensional vector space.

Let $\mathcal{B} = \{\mathbf{x}_1, \dots, \mathbf{x}_n\}$ and $\mathcal{C} = \{\mathbf{y}_1, \dots, \mathbf{y}_n\}$ be bases of V. Let $\mathrm{id}: V \to V$ be the identity function.

- The **transition matrix** matrix $[id]_{\mathcal{B}}^{\mathcal{C}}$ is the $n \times n$ matrix whose j^{th} column is the vector $[\mathbf{x}_j]_{\mathcal{C}}$.
- Theorem 4.30.1: For all $\mathbf{x} \in V$, we have $[\mathrm{id}]_{\mathcal{B}}^{\mathcal{C}}[\mathbf{x}]_{\mathcal{B}} = [\mathbf{x}]_{\mathcal{C}}$.



• Theorem 4.30.2: The matrix $[id]_{\mathcal{B}}^{\mathcal{C}}$ is invertible, and $([id]_{\mathcal{B}}^{\mathcal{C}})^{-1} = [id]_{\mathcal{C}}^{\mathcal{B}}$.



Matrix Representations of Linear Transformations

Let *V* and *W* be finite dimensional vector spaces.

Let $\mathcal{B} = \{\mathbf{x}_1, \dots, \mathbf{x}_n\}$ be an ordered basis of V, and let $\mathcal{C} = \{\mathbf{y}_1, \dots, \mathbf{y}_m\}$ be an ordered basis of W. Let $f: V \to W$ be a linear transformation.

- We define $[f]_{\mathcal{B}}^{\mathcal{C}}$ to be the matrix whose columns are $[f(\mathbf{x}_1)]_{\mathcal{C}}, [f(\mathbf{x}_2)]_{\mathcal{C}}, \dots [f(\mathbf{x}_n)]_{\mathcal{C}}.$
- **Theorem 4.37:** With the above notation, for all $\mathbf{x} \in V$, we have

$$[f]_{\mathcal{B}}^{\mathcal{C}}[\mathbf{x}]_{\mathcal{B}} = [f(\mathbf{x})]_{\mathcal{C}}.$$

$$V \xrightarrow{f} W$$

$$[\cdot]_{\mathcal{B}} \downarrow \qquad \qquad \downarrow [\cdot]_{\mathcal{C}}$$

$$\mathbb{R}^{n} \xrightarrow{[f]_{\mathcal{B}}^{\mathcal{C}}} \mathbb{R}^{m}$$

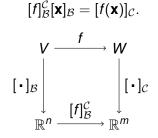
Matrix Representations of Linear Transformations

Let *V* and *W* be finite dimensional vector spaces.

Let $\mathcal{B} = \{\mathbf{x}_1, \dots, \mathbf{x}_n\}$ be an ordered basis of V, and let $\mathcal{C} = \{\mathbf{y}_1, \dots, \mathbf{y}_m\}$ be an ordered basis of W.

Let $f: V \to W$ be a linear transformation.

- We define $[f]_{\mathcal{B}}^{\mathcal{C}}$ to be the matrix whose columns are $[f(\mathbf{x}_1)]_{\mathcal{C}}, [f(\mathbf{x}_2)]_{\mathcal{C}}, \dots [f(\mathbf{x}_n)]_{\mathcal{C}}.$
- Theorem 4.37: With the above notation, for all $\mathbf{x} \in V$, we have



Theorem 4.39: Let V be a finite dimensional vector space, with ordered bases \mathcal{B} and \mathcal{C} . Let $f: V \to V$ be a linear transformation, and let $P = [\mathrm{id}]_{\mathcal{C}}^{\mathcal{B}}$. Then

$$[f]_{\mathcal{C}}^{\mathcal{C}} = P^{-1}[f]_{\mathcal{B}}^{\mathcal{B}}P.$$

